NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION: Bachelor of science in Applied Mathematics and Statistics	
QUALIFICATION CODE: 35BAMS	**LEVEL:** 7
COURSE CODE: NUM702S	**COURSE NAME:** NUMERICAL METHODS 2
SESSION: NOVEMBER 2019	**PAPER:** THEORY
DURATION: 3 HOURS	MARKS: 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER	Dr S.N. NEOSSI NGUETCHUE
MODERATOR:	Prof S.S. MOTSA

INSTRUCTIONS

1. Answer ALL the questions in the booklet provided.
2. Show clearly all the steps used in the calculations. All numerical results must be given using 4 to 5 decimals where necessary unless specified otherwise.
3. All written work must be done in blue or black ink and sketches must be done in pencil.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

Attachments

None

Problem 1 [32 Marks]

1-1. Find the MacLaurin expansion of the function $f(x) = \dfrac{1}{\sqrt{1-x}}$ about $x_0 = 0$. [5]

1-2. Establish that the Padé approximation $R_{2,2}(x)$ for $f(x) = \dfrac{1}{\sqrt{1-x}}$ expanded about $x_0 = 0$ is given by [15]

$$R_{2,2}(x) = \frac{16 - 12x + x^2}{16 - 20x + 5x^2}.$$

1-3. Compare the following approximations to $f(x) = \tan(x)$ [12]

$$\begin{aligned} \text{Taylor: } T_9(x) &= 1 + \frac{x}{3} + \frac{2x^2}{15} + \frac{17x^3}{315} + \frac{62x^4}{2835} \\ \text{Padé: } R_{5,4}(x) &= \frac{945x - 105x^3 + x^5}{945x - 420x^2 + 15x^4} \end{aligned}$$

on the interval $[0, 1.4]$ using 8 equally spaced points x_k with $h = 0.2$. Your results should be correct to 7 significant digits.

Problem 2 [34 Marks]

2-1. What is an orthogonal polynomial and what is the importance of orthogonal polynomials in least-squares problems? [5]

2-2. Show that Chebyshev polynomials $(T_k)_{k \geq 0}$, where $T_k(x) = \cos\left[k \cos^{-1}(x)\right]$ for $x \in [-1, 1]$, are orthogonal with respect to an appropriate inner product $< ., . >$ to be defined. [9]

2-3. Determine the Chebyshev series expansion of $f(x) = \arccos(x)$ in the form. [10]

$$f(x) \sim {\sum_{k=0}^{\infty}}' c_k T_k(x) = \frac{1}{2} c_0 T_0(x) + c_1 T_1(x) + c_2 T_2(x) + \cdots$$

where $c_k = < f, T_k > / < T_k, T_k >$ for $k \geq 1$ and $c_0/2 = < f, T_0 > /\pi$, $< ., . >$ being the inner product alluded to in **2-2.**

2-4. Find the Fourier series of $f(x) = \sin(x), \quad x \in [0, \pi]$. [10]

Problem 3 [34 Marks]

3-1. Given the integral

$$\int_{0.04}^{1} \frac{1}{\sqrt{x}}\, dx = 1.6.$$

Compute $T(J) = R(J, 0)$ for $J = 0, 1, 2$ using the recursive trapezoidal rule. [9]

3-2. State the three-point Gaussian Rule for a continuous function f on the interval $[-1, 1]$ and show that the rule is exact for $f(x) = x^4 + 3$. [5]

3-3. The matrix A and its inverse are A^{-1} are given below

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}, \qquad A^{-1} = -\frac{1}{4} \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}.$$

• Use a related power method to find the eigenvalue of the matrix A with the smallest absolute value and the associated eigenvector. Start with the vector $\mathbf{x}^{(0)} = (1, 0, 0)^T$ and perform three iterations. [10]

3-4. Assume A is a symmetric matrix and we want to compute all its eigenvalues. [5]
Explain what are Householder's and QR methods and how they can be used for this purpose.

3-5. Let $w \in \mathbb{R}^n$ be a vector such that $w^T w = 1$. Define the Householder's matrix associated with w and show that it is symmetric and orthogonal. [5].

God bless you !!!